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TWO-PHASE FILTRATION IN MIXED-WETTABLE POROUS MEDIA

A. V. Domanskii and V. I. Pen'kovskii

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The pressure difference

$$p - p_1 = p_c, \tag{0.1}$$

called the "capillary pressure at the phase boundary," is the governing relation which closes the standard [1] system of filtration equations for immiscible incompressible fluids. This system is composed of the equations of motion (generalized Darcy's law)

$$v = -kf(s)\partial p/\partial x,\ v_1 = -k_1 f_1(s)\partial p_1/\partial x \tag{0.2}$$

and the mass conservation law

$$m\partial s/\partial t + \partial v/\partial x = 0,\ -m\partial s/\partial t + \partial v_1/\partial x = 0. \tag{0.3}$$

Here, p is the pressure in the fluid (oil, for example), which occupies the fraction s of the pore space; $s_1 = 1 - s$ is the saturation of the second fluid (water), with a pressure p_1; k and k_1 are the total permeabilities of the medium, referred to the viscosities, at s = 1 and s = 0, respectively; f and f_1 are the relative ("phase") permeabilities; v and v_1 are the rates of filtration of the fluids; m is porosity; x is a coordinate; t is time.

The capillary pressure $p_c = p_c(s)$ is determined experimentally under static conditions and is assigned in the form of a fixed function of saturation s. In particular, the medium is assumed to be hydrophilic at $p_c \geq 0$ and hydrophobic at $p_c \leq 0$. However, the assumption that the sign of p_c in (0.1) is fixed is not always valid in problems of oil-field mechanics, since the problem of the wettability of the rocks which make up the oil-bearing strata cannot always be solved unambiguously [2, 3].

It was suggested in [4] that there are three main classes of porous media with regard to the case of two immiscible fluids saturating these media: 1) wettable ($p_c \geq 0$); 2) unwettable ($p_c \leq 0$); 3) intermediate-wettable or "mixed-wettable" [5]. A fluid–fluid–porous-medium system of the third type is characterized by a change in the sign of the function p_c.

As is known [6], the wettability of a system under static conditions is determined by the contact angle θ from the Young equilibrium equation

$$\cos\theta = (\gamma_1 - \gamma_2)/\gamma_{1,2}, \tag{0.4}$$

where γ_i (i = 1, 2) are the specific free energies of the interfaces between the skeleton and each of the fluids; $\gamma_{1,2}$ is the specific free energy of the interface between the fluids (surface tension). If $\gamma_1 > \gamma_2$, then the angle θ is acute, and fluid 2 wets the solid more readily than fluid 1. When $\gamma_1 \approx \gamma_2$, $\cos\theta \approx 0$, the fluids wet the solid equally well. The relationship between the values of γ_i may change over time even under static conditions

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("meniscus aging" effect). Significantly larger changes in the specific free energies are seen during motion [6]. Thus, the capillary pressure which figures in governing relation (0.1) should be regarded as a dynamic characteristic of the fluid−fluid−porous-medium system.

Here, we propose one possible variant of assigning the dependence of dynamic capillary pressure on saturation and we look at an example of the flow of two fluids in a mixed-wettable porous medium. The results of calculations performed by the proposed algorithm are compared with experimental data.

1. Capillary Pressure in Mixed-Wettable Porous Media. We will first examine Poiseuille flow in a thin circular capillary tube of the radius r_c. The equation of the line of the particles $z = z(r, t)$ at the initial moment of time $t = 0$ perpendicular to the z axis of the tube has the form $z(r, t) = \langle v\rangle t(1 - 2r^2/r_c^2)$ in the coordinate system moving the mean velocity $\langle v\rangle$. Calculating the mean curvature $\langle K\rangle$ of this line, we obtain

$$\langle K(t)\rangle = \frac{1}{r_c}\int_0^{r_c} \frac{|z''_{rr}|}{(1+z'^2_r)^{3/2}}\,dr = \frac{1}{r_c}\,\frac{l(t)}{\sqrt{l^2+1/16}},$$

from which it is evident that the radius of mean curvature of the line of the particles rapidly approaches the radius of the tube r_c with an increase in the dimensionless displacement $\ell(t) = \langle v\rangle t/r_c$.

On the other hand, let $\gamma_1 \approx \gamma_2$ in Eq. (0.4) and let there be an interface (meniscus) S between the two immiscible fluids moving in the tube at the prescribed mean velocity $\langle v\rangle =$ const. Let μ_i be viscosity, ρ_i be the densities of the fluids ($i = 1, 2$), αp_γ be the fraction of the capillary pressure jump acting on the first fluid, and $-(1 - \alpha)p_\gamma$ be the fraction of pressure acting on the second fluid ($0 < \alpha < 1$, while the total pressure jump $\alpha p_\gamma - [-(1 - \alpha)p_\gamma] = p_\gamma$ is equal to the capillary pressure). The pressure in the first fluid, to the left of the meniscus, is equal to $p_s + \alpha p_\gamma$. To the right, in the second fluid, $p - (1 - \alpha)p_\gamma$ (p_s is the hydrodynamic pressure in the neighborhood of the meniscus). In a coordinate system moving at the same velocity as the meniscus $\langle v\rangle$, we distinguish two cross sections S_1 and S_2. These sections are located sufficiently far from the meniscus. We take the below equation, expressing the theorem of the momentum change [7], and we apply it to the volumes of the fluids τ_i bounded by the lateral surfaces S^i, the sections S_i, and the surface of the meniscus S

$$\frac{d}{dt}\int_{\tau_i} v_{zi}\rho_i\,d\tau - \int_{\Sigma} v_{zi}\rho_i v_{ni}\,dS = F_{zi}$$
$$(\Sigma = S^i \cup S_i \cup S). \tag{1.1}$$

Here, F_{zi} is the projection of the principal vector of the external forces applied to these volumes; v_{zi} is the projection of the relative velocities on the z axis, which is directed along the axis of the capillary tube; v_{ni} is the projection of the velocities on a normal to the surfaces. The flow of the fluids is close to Poiseuille flow outside a certain neighborhood of the meniscus, so it is natural to assume that the forces of viscous friction acting over the lateral surfaces S^i are balanced by the gradients of the hydrodynamic pressures $p_s - p_1$ and $p_2 - p_s$ (p_i is the pressure in the section S_i).

Noting that $v_{ni} = 0$ on S and S^i, that $v_{zi} = v - \langle v\rangle = \langle v\rangle(1 - 2r^2/r_c^2)$ on S_i, and that the projection of the principal vector of the force corresponding to the jump in capillary pressure p_γ is equal to $2\pi r_c\gamma_{1,2}\cos\theta$ (θ is the angle which is formed by the meniscus with the surface of the tube), we use Eq. (1.1) to obtain the relation $-\pi\rho_1 r_c^2\langle v\rangle^2/3 = -\alpha 2\pi r_c\gamma_{1,2} \times \cos\theta$, $\pi\rho_2 r_c^2\langle v\rangle^2/3 = (1 - \alpha)2\pi r_c\gamma_{1,2}\cos\theta$. Excluding $\alpha = \rho_1/(\rho_1 + \rho_2)$, we find the radius $R_c = r_c/\cos\theta$ of curvature of the meniscus in the form

$$R_c = 6\gamma_{1,2}/[\langle v\rangle^2(\rho_1 + \rho_2)]. \tag{1.2}$$

As might be expected, the kinematic action of the particles of the fluids on the surface of the meniscus is proportional to the square of the mean velocity. Since filtration flows are classed as "creeping" flows and are characterized by velocities v which are 2-4 times less than the value given by Eq. (1.2) for $\langle v\rangle = \langle v\rangle_{\max} = \sqrt{6\gamma_{1,2}/(r_c(\rho_1 + \rho_2))}$, it can be assumed that, in a porous medium, the curvature of the meniscus is independent of the filtration velocity and is determined mainly by the geometry of the pore space and the

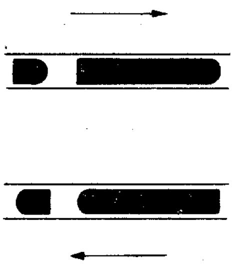

Fig. 1

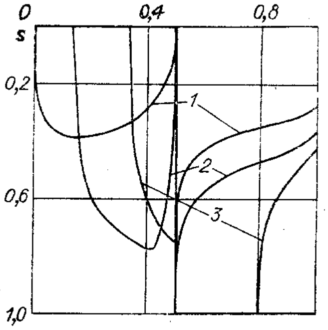

Fig. 2

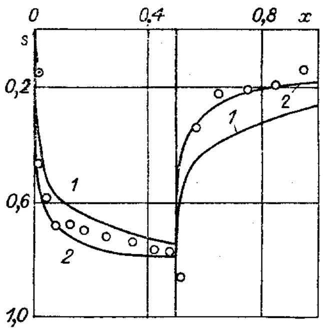

Fig. 3

physicochemical reactions between the fluids and the skeleton of the medium during their mutual displacement.

Figure 1 shows a water-filled quartz capillary tube with an occlusion of crude oil of the radius $r_c = 0.3\cdot10^{-3}$ m. The arrows denote the directions of movements of the oil. It is evident that the curvature of the meniscus changes in relation to the direction of motion.

When the interface between the fluids is displaced, one fluid is separated from the solid surface (cohesion) while the other is simultaneously attached to it (adhesion). The difference between the energies expended on detachment and attachment of the respective fluids should be compensated for by the work done in displacing the meniscus, i.e., the work done in overcoming capillary pressure. Since the energy of cohesion is greater than the energy of adhesion – at least for a rock–oil–water system – then the capillary pressure resulting from motion will prevent displacement from taking place. In other words, the water-filled pore channel will behave as if it were hydrophilic, while the oil-filled channel will behave as though it were hydrophobic. This provides grounds for suggesting that, for mixed-wettable porous media, the sign of the function p_c in Eq. (0.1) coincides with the sign of the derivative $\partial s/\partial t$. Here, the hysteresis of capillary pressure is greater here than in wettable media such as in systems of the type medium–liquid–gas. However, the problem of constructing primary and secondary curves of the developed surfaces is simpler, since there is a priority for filling of the pore channels: the displacing ("nonwetting") fluid, moving under the influence of hydrodynamic pressure along the paths of least resistance, tends to penetrate the relatively coarse pores. A similar priority for filling of pores by gas (air) in the drainage cycle exists in wettable media – for which experimental methods of determining static capillary pressure $p_k(S)$ are well-established.

We will assume that the primary curve of the hysteresis of capillary pressure

$$p_c = \varkappa\varphi(s)\ (s(x,\ 0) = 0,\ \partial s/\partial t > 0), \tag{1.3}$$

where $\varkappa > 0$, depends on the difference in the specific energies of adhesion of oil and cohesion of water on the surface of particles of the skeleton of the porous medium; $\varphi(s)$ is a dimensionless function analogous to the Leverett function. Then the second part of the primary curve of the hysteresis loop is represented in the form

$$p_c = -\varkappa_1\varphi_1(s)\ (s(x,\ 0) = 1,\ \partial s/\partial t < 0). \tag{1.4}$$

Here, $\varkappa_1$ is a parameter determined by the difference in the specific energies of adhesion of water and cohesion of oil, while the functions $\varphi_1(s) = \varphi(s_1) = \varphi(1 - s)$, since the order of the filling of pores with water in a hydrophobic medium is similar to the order of filling of pores with oil in a hydrophilic medium with the same structure. In a state of rest ($v = v_1 = 0$), pressure is equalized in the fluids ($p_c = 0$). In accordance with this, if the displacement process continued from the state of rest beginning with the moment of time $t = t_0$, then the capillary pressure developed as a result of motion will be the difference

$$p_c = \varkappa[\varphi(s) - \varphi(s^0)],\ s_t(x,\ t_0 + 0) > 0,\ s_t(x,\ t_0 - 0) > 0$$

between the specific energies expended and already expended. Similarly, the secondary curves of the reserve process of "final displacement" will take the form

$$p_c = -\varkappa_1[\varphi(s_1) - \varphi(s_1^0)],\quad s_t(x,\ t_0 + 0) > 0,$$
$$s_t(x,\ t_0 - 0) > 0\ (s^0 = s(x,\ t_0)).$$

In the case of a change in the sign of displacement, it follows from consideration of the "mutuality of the expended energies" that

$$p_{\mathrm{c}} = \begin{cases} \varkappa\left[\varphi(s_1) - \varphi(s_1^0)\right], & s_t(x, t_0 + 0) > 0, \quad s_t(x, t_0 - 0) < 0, \\ -\varkappa_1[\varphi(s) - \varphi(s^0)], & s_t(x, t_0 + 0) < 0, \quad s_t(x, t_0 - 0) > 0. \end{cases}$$

If the forces of interaction between the skeleton of the medium and the water are greater, then $\varkappa > \varkappa_1$. When $\varkappa_1 = \varkappa_1(t) \to 0$, spontaneous capillary impregnation of the medium with water may occur [8]. However, the change in the parameters $\varkappa$ and $\varkappa_1$ over time will evidently be of a diffusional character and will be related to the slow process of redistribution of "free sites" on the surface of particles of the medium.

2. Numerical Solution. Experimental Data. Let the initial distribution of oil saturation

$$s^0 = s(x, 0) = \begin{cases} 0, & 0 < x < x_1, \\ 1, & x_1 < x < 1 \end{cases} \tag{2.1}$$

model a plugged bed of unit length into which water began to be pumped with a prescribed pressure gradient Δp at $t > 0$ over the section $x = 1$. The boundary conditions of the displacement problem are:

$$t > 0,\ x = 0\colon\ p = p_1 = 0,\ x = 1\colon\ p_1 = \Delta p,\ v = 0. \tag{2.2}$$

In the unidimensional case with capillary pressure functions assigned in the form (1.3) or (1.4), system (0.1)-(0.3) reduces to a single equation for saturation

$$\frac{m\partial s}{\partial t} = \frac{\partial}{\partial x}\left(a^{\pm}(s)\frac{\partial s}{\partial x} + V(t)\,b(s)\right) \equiv \frac{\partial\sigma}{\partial x}. \tag{2.3}$$

Here, $a^{\pm} = kk_1ff_1Rdp_{\mathrm{c}}^{\pm}/ds$; $p_{\mathrm{c}}^{-} = \varkappa\varphi(s)$; $p_{\mathrm{c}}^{+} = -\varkappa_1\varphi_1(s)$; $b = k_1f_1R$; $R \equiv (kf + k_1f_1)^{-1}$.

We designate $D^- = \{0 \leqslant x \leqslant x_1\}$, $D^+ = \{x_1 \leqslant x \leqslant 1\}$. The boundary conditions on the external boundaries (2.2) will take the form

$$t > 0,\ x = 0\colon\ s = 0,\ x = 1\colon\ a^+\partial s/\partial x + Vb = V. \tag{2.4}$$

The compatibility conditions for the flows and phase pressure at the point x_1 with simultaneous motion of the oil and water lead us to the relations

$$\begin{gathered} (a^+\partial s/\partial x + Vb)|_{x=x_1+0} = (a^-\partial s/\partial x + Vb)|_{x=x_1-0}, \\ s^+ \equiv s(x_1 + 0,\ t) = 1,\ s^- \equiv s(x_1 - 0,\ t) = 0. \end{gathered} \tag{2.5}$$

Until the oil displacement front reaches the point x_1, at $x = x_1$ we have the equality

$$(a^-\partial s/\partial x + Vb)|_{x=x_1-0} = 0, \quad s^+ = 1. \tag{2.6}$$

As the oil displacement front approaches the point x_1, at the moment $t_0 - 0$ the pressure p_1 in the water phase undergoes a discontinuity $p_1(x_1 + 0, t_0 - 0) - p_1(x_1 - 0,\ t_0 - 0) = \varkappa_1\varphi_1(1) + \varkappa\varphi(s(x_1 - 0,\ t_0 - 0)) = \varkappa\varphi(s(x_1 - 0,\ t_0 - 0))$.

We obtain the following functional relation from conditions (2.4)-(2.6) to determine $V(t)$

$$V(t) = -(\Delta p - F(s^-) + F_0(s(1, t)))/J(t), \tag{2.7}$$

where $F(s) = \int\limits_0^s k_1f_1R\frac{dp_{\mathrm{c}}^-}{ds}\,ds$; $F_0(s) = \int\limits_s^1 k_1f_1R\frac{dp_{\mathrm{c}}^+}{ds}\,ds$; $J = \int\limits_0^1 R(s)\,dx$.

Thus, the initial problem consists of finding the solution of Eqs. (2.3) – where $V(t)$ is found from Eq. (2.7) – in the regions D^+, D^- with boundary conditions (2.4), initial conditions (2.1), and compatibility conditions (2.5), (2.6).

Existing methods of numerically solving problems of two-phase filtration in regions with permeability and saturation discontinuities (such as for "suspended" discontinuities) generally involve the use of explicit difference schemes [9, 10] or are based on "blurring" of these discontinuities [11].

With allowance for the specifics of the problem formulated above, we chose to use an unconditionally stable implicit difference scheme involving "splitting" of the entire filtration region into subregions which were uniform with respect to the physical properties of the substances. We then constructed a special iteration process. Finite-difference approximation of Eq. (2.3) was accomplished by the integrodifferential method [12]. The approximation has the form

$$\begin{gathered} m\frac{s_i^{j+1}-s_i^j}{\tau}=\frac{\sigma_{i+1/2}^{j+1}-\sigma_{i-1/2}^{j+1}}{h},\quad 1\leqslant i\leqslant N, \\ \sigma_{i+1/2}^{j+1}=(a^{\pm})_{i+1/2}^{j+1}\frac{s_{i+1}^{j+1}-s_i^{j+1}}{h}+V^{j+1}b_{i+1/2}^{j+1}. \end{gathered} \tag{2.8}$$

The order of approximation of Eq. (2.8) is $O(\tau + h^2)$.

The boundary condition at x = 1 from (2.4) was also approximated with the second order with respect to h, similar to the case in [12]:

$$m\frac{s_{N-1}^{j+1}-s_{N-1}^j}{\tau}=\frac{V^{j+1}-\sigma_{N-1/2}^{j+1}}{h}. \tag{2.9}$$

The values of s^- were referred to the node immediately to the left of the node of the discontinuity $x_I = x_1$. Thus, the values of saturation to the left and right of the discontinuous point were placed in different storage cells of the computer. As a result, the compatibility conditions (2.5), (2.6), written in difference form

$$\sigma_{I-1/2}^{j+1}=\sigma_{I+1/2}^{j+1},\quad s_{I-1}^{j+1}=0,\quad s_I^{j+1}=1,\quad \sigma_{I-1/2}^{j+1}=0,\quad s_I^{j+1}=1,$$

can be regarded as the boundary conditions for the "inlet" boundary of the region D^-. Approximating them with second-order accuracy with respect to h, similar to (2.9), we find that

$$m\frac{s_{I-1}^{j+1}-s_{I-1}^j}{\tau}=\frac{\sigma_{I+1/2}^{j+1}-\sigma_{I-3/2}^{j+1}}{h},\quad s_{I-1}^{j+1}=0; \tag{2.10}$$

$$m\frac{s_{I-1}^{j+1}-s_{I-1}^j}{\tau}=-\frac{\sigma_{I-3/2}^{j+1}}{h},\quad s_I^{j+1}=1. \tag{2.11}$$

Initial condition (2.1) and boundary condition (2.4) are written in the form

$$s_i^0=0,\quad 0\leqslant i\leqslant I-1,\quad s_i^0=1,\quad I\leqslant i\leqslant N; \tag{2.12}$$

$$s_0^j=0. \tag{2.13}$$

Thus, boundary-value problem (2.1), (2.3)-(2.7) was solved by finite-difference scheme (2.8)-(2.13). Since the problem is nonlinear, we calculated saturation by the iterative method. The values of the coefficients of the equation and the functional V(t) were taken from the previous iteration [the integrals in (2.7) were calculated from the trapezoid formula]. Here, in order to "split" the problem, the value of the flow on the right $\sigma_{I+1/2}^{j+1}$ in (2.10) was also determined from the previous iteration.

We thus solved the same type of problem in each iteration for the regions D^-, D^+ – the flow of the displacing phase is assigned at the inlet, while saturation is assigned at the outlet. It is clear from the foregoing that the algorithm is organized so that the problem can actually be solved by the method of three-point trial run. Here, the coefficients at the nodes x_{I-1}, x_I are redefined.

We used the following dependences of capillary pressure and relative permeability on saturation in the calculations: $f(s) = f_1(1-s) = s^{3.5}$, $\varphi(s) = \varphi_1(1-s) = (s/(0.85-s))^{0.5}$, $k/k_1 = 6.31$, $\varkappa = \varkappa_1$, $\Delta p^0 = \Delta p/\varkappa = 3$, $x_1 = 0.5$. The iteration was continued until satisfaction of the condition $\max\limits_i |s_i^{q+1} - s_i^q| < 10^{-3}$ (q is the number of the iteration). The time step τ was chosen on the conditions of convergence of the iterations with a specified accuracy and satisfaction of the material-balance condition to within 5%.

Some of the results are shown in Fig. 2, where line 3 represents the distribution of saturation up to the moment the oil displacement front reaches the point x_1. Line 2 shows the same at this moment, while line 1 shows the distribution at the next moment.

Let us determine possible values of the physial parameters of the problem for a system composed of fine-grained sand, kerosine, and water. Tests conducted in a quartz capillary tube with the radius $0.3\cdot10^{-3}$ m show that the maximum (in the case of a receding meniscus) height of capillary rise of water in the tube $h_w \approx 4.9\cdot10^{-2}$ m, which is evidence of nearly complete wettability of the inside surface of the tube (the surface tension of water $\gamma_w \approx 73.10^{-3}$ N/m). The height of capillary rise of the kerosine in the tube $h_k \approx 2.8\cdot10^{-2}$ m. At the same time, the maximum height of the column of water supported by capillary forces with an overlying layer of kerosine of the thickness h_k is $h_{wk} \approx 1.5 \cdot 10^{-2}$m. Using the Jurin–Laplace formula, for the given h_{wk} we obtain $\gamma_{wk} \cos \theta_{wk} \approx 22 \cdot 10^{-3}$ N/m (γ_{wk} is the surface tension at the water–kerosine boundary and θ_{wk} is the contact angle). Thus, the interfacial specific free energy in the capillary–water kerosine system is $22/73 \approx 0.3$ of that developed in the capillary–water–air system. If we use the function $p_w(s_w) = 0.5[s_w/(0.85 - s_w)]^{0.5}$ to approximate the empirical dependence of capillary pressure p_w on saturation s_w with air for fine-grained sand of the porosity $m = 0.34$ and a filtration coefficient (for water) $k_1 = 8.6$ m/day, then in (1.3) for a sand–kerosine–water system we can take $\varkappa = 0.5 \cdot 0.3 = 0.15$ m, $\varphi(s) = [s/(0.85 - s)]^{0.5}$ (s is the saturation of the sand with kerosine), $\varkappa_1 = 0.1$ m.

Figure 3 compares calculated results (curves 1 and 2) with empirical data (points) for the moment the water phase meets the boundary of the wetting properties of the specimen. The tests were conducted on a tube 1 m long and 0.02 m in diameter filled with fine-grained sand. The working fluids were a solution of sodium chloride with a concentration of 30 g/liter and kerosine. The gradient of the levels at the ends of the tube was 1 m water column. The saturation of the pore space with the water phase was determined by the electrical method. Curve 1 shows the results of calculations performed by the above-described numerical scheme, while curve 2 shows the approximate analytical solution of the problem obtained by the well-known method of successive substitution of steady states. The calculated results agree at least qualitatively with the experimental data. Since the more viscous fluid is displaced by the less viscous fluid in the subregion (0.5, 1), there was some instability in the formation of the saturation profile in the experiments. This caused the empirical scatter to be greater than in the region (0, 0.5).

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